12. Consider the following linear programming problem:

Max $3 A+3 B$
s.t.
$2 A+4 B \leq 12$
$6 A+4 B \leq 24$
$A, B \geq 0$
a. Find the optimal solution using the graphical solution procedure.
b. If the objective function is changed to $2 A+6 B$, what will the optimal solution be?
c. How many extreme points are there? What are the values of $A$ and $B$ at each extreme point?
20. For the linear program

Max $3 A+2 B$
s.t.
$A+B \geq 4$
$3 A+4 B \leq 24$
$A \geq 2$
$A-B \leq 0$
$A, B \geq 0$
a. Write the problem in standard form.
b. Solve the problem.
c. What are the values of the slack and surplus variables at the optimal solution?
41. Southern Oil Company produces two grades of gasoline: regular and premium. The profit contributions are $\$ 0.30$ per gallon for regular gasoline and $\$ 0.50$ per gallon for premium gasoline. Each gallon of regular gasoline contains 0.3 gallons of grade A crude oil and each gallon of premium gasoline contains 0.6 gallons of grade A crude oil. For the next production period, Southern has 18,000 gallons of grade A crude oil available. The refinery used to produce the gasolines has a production capacity of 50,000 gallons for the next production period. Southern Oil's distributors have indicated that demand for the premium gasoline for the next production period will be at most 20,000 gallons.
a. Formulate a linear programming model that can be used to determine the number of gallons of regular gasoline and the number of gallons of premium gasoline that should be produced in order to maximize total profit contribution.
b. What is the optimal solution?
c. What are the values and interpretations of the slack variables?
d. What are the binding constraints?
45. Consider the following linear program:
$\operatorname{Max} 1 A-2 B$
s.t.
$-4 A+3 B \leq 3$
$1 A-1 B \leq 3$
$A, B \geq 0$
a. Graph the feasible region for the problem.
b. Is the feasible region unbounded? Explain.
c. Find the optimal solution.
d. Does an unbounded feasible region imply that the optimal solution to the linear program will be unbounded?

